SUPERSONIC FLOW AROUND THE TRAILING EDGE OF A THIN PROFILE

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The method of mergeable asymptotic expansions has recently been used effectively in investigations devoted to the study of boundary layer interaction with an external inviscid flow at high subcritical Reynolds numbers Re. The asymptotic analysis permits obtaining a limit pattern of the flow around a solid as Re $\to \infty$, and determining the similarity and quantitative regularity laws which are in good agreement with experimental results. Thus by using the method of mergeable asymptotic expansions it is shown in [1-4] that near sites with high local curvature of the body contour and flow separation and attachment points, an interaction domain appears that has a small length on the order of $\mathrm{Re}^{-3/8}$. In this flow domain, which has a three-layer structure, the pressure distribution in a first approximation already depends on the change in boundary-layer displacement thickness, while the induced pressure gradient, in turn, influences the flow in the boundary layer. An analogous situation occurs in the neighborhood of the trailing edge of a flat plate where an interaction domain also appears [5, 6]. The flow in the neighborhood of the trailing edge of a flat plate around which a supersonic viscous gas flows was examined in [7]. Numerical results in this paper show that the friction stress on the plate surface remains positive everywhere in the interaction domain, and grows on approaching the trailing edge. The supersonic flow around the trailing edge of a flat plate at a small angle of attack was investigated in [8, 9]. Supersonic flow of a viscous gas in the neighborhood of the trailing edge of a flat plate at zero angle of attack is examined in [10], but with different velocity values in the inviscid part of the flow on the upper and lower sides of the plate. The more general problem of the flow around the trailing edge of a profile with small relative thickness is investigated in this paper.

Let us consider the flow around a thin profile at a small angle of attack by a uniform supersonic viscous gas stream as the characteristic Reynolds number tends to infinity (Re = $\rho_\infty u_\infty l/\mu_\infty = \varepsilon^{-2}$, where ρ_∞, u_∞, μ_∞ are the density, velocity, and dynamic viscosity coefficient of the incoming stream, and l is the profile chord length). Henceforth, only dimenless quantities are used, whereupon all the linear dimensions are referred to l, the velocities to u_∞, the densities to ρ_∞, the pressure to $\rho_\infty u_\infty^2$, the enthalpy to u_∞^2, and the dynamic viscosity coefficient to μ_∞. We shall use a Cartesian X_0, Y_0 coordinate system with $0X_0$ axis along the incoming flow velocity direction and origin at the profile trailing edge for the description of the flow around the profile. Let the slopes of the profile surface to the incoming stream direction be of the order $\tau \ll 1$. The equation of the profile surface is written in the Cartesian coordinates in the form

$$Y_0 = \tau F_{1,2}(X_0) \quad \text{for} \quad -1 \leqslant X_0 \leqslant 0,$$

where $F_1(X_0)$ and $F_2(X_0)$ are the shapes of the upper and lower profile surfaces, respectively, and τ is a small parameter characterizing the relative profile thickness and is independent of the Reynolds number. Then the flow around the profile will be described by the linear theory of supersonic flows that yields the following expressions for the pressure and the longitudinal velocity component on the upper and lower profile surfaces:

$$p = \frac{1}{\gamma M_\infty^2} \pm \frac{\tau}{\sqrt{M_\infty^2 - 1}} \frac{dF_{1,2}}{dX_0}, \quad u = 1 \mp \frac{\tau}{\sqrt{M_\infty^2 - 1}} \frac{dF_{1,2}}{dX_0}, \tag{1}$$

where γ is the ratio of the specific heats, and M_∞ is the free stream Mach number.

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We make the following additional assumptions relative to the profile shape. Let the profile surface shape be such that in a certain neighborhood of the profile trailing edge with $X_0 \sim \varepsilon^{3/4}$, the slopes of its upper and lower surfaces are of the order $\varepsilon^{1/2} = \mathrm{Re}^{-1/4}$. Since the parameter τ is independent of Re, and therefore cannot tend to zero as $\mathrm{Re} \to \infty$, the profile shape should be selected so that, firstly, a smooth passage from slopes on the order of τ on the main part of the surface with $X_0 \sim 1$ to angles near the trailing edge, on the order of $\varepsilon^{1/2}$, should be assured, and secondly, separation-free flow around the profile surface should be realized up to at least the neighborhood of its trailing edge with $X_0 \sim \varepsilon^{3/4}$.

Under the assumptions made relative to the slopes of the profile contour in the neighborhood of the trailing edge, the shape of the profile surface should satisfy the condition $dF_{1,2}/dX_0 = 0$ for $X_0 = 0$. The zero streamline coincides with the profile surface for $-1 \leqslant X_0 \leqslant 0$, where the condition of impermeability of the contour is imposed. In order to assure compliance with the adhesion condition, a boundary layer must be introduced on the upper and lower sides of the profile. The flow in the boundary layer is described by the usual equations for a compressible boundary layer with pressure and external velocity distributions governed by (1), and a given temperature surface. By integrating these equations velocity and enthalpy profiles, friction stress and heat flux distributions over the profile surface, and, in particular, the friction stress on the upper and lower sides of the profile trailing edge can be obtained. In contrast to the case investigated in [8, 9] of the supersonic flow around a flat plate at an angle of attack, these friction stresses can differ in magnitude from each other. However, the different friction stresses on the trailing edge do not appear in this case because the inviscid flow on the upper and lower sides has different values of the velocity (see [10]), but because the velocity distribution on the outer boundary-layer boundary, and the pressure gradient at different sides of the profile, may differ substantially from each other. In this case a boundary-layer interaction domain with the outer inviscid supersonic flow [4] is realized in the neighborhood of the profile trailing edge in this case. It is convenient to use an orthogonal (x, y) coordinate system coupled to the zero streamline and with origin at the profile trailing edge to describe the flow in this domain. As in [1-4, 8-10], the interaction domain is divided into domains with different coordinate scales and flow functions: Domain 1 corresponds to the perturbed inviscid supersonic flow zone ($x \sim y \sim \varepsilon^{3/4}$), domain 2 is the main part of the boundary layer ($y \sim \varepsilon$), and domain 3 is the viscous sublayer in which the velocity perturbation is of the order of the velocity itself ($y \sim \varepsilon^{5/4}$). By using the stream function ψ, the asymptotic expansions of the coordinates and the stream function in domain 1 can be represented in the form

$$x = \varepsilon^{3/4} x_1, \quad \psi = \varepsilon^{3/4} \psi_1, \quad p = \frac{1}{\gamma M_\infty^2} + \varepsilon^{1/2} p_1(x_1, \psi_1) + \ldots,$$

$$\rho = 1 + \varepsilon^{1/2} \rho_1(x_1, \psi_1) + \ldots, \quad u = 1 + \varepsilon^{1/2} u_1(x_1, \psi_1) + \ldots, \quad v = \varepsilon^{1/2} v_1(x_1, \psi_1),$$

$$h = \frac{1}{(\gamma - 1) M_\infty^2} + \varepsilon^{1/2} h_1(x_1, \psi_1) + \ldots, \quad y = \varepsilon^{3/4} \psi_1 + \varepsilon^{5/4} y_1(x_1, \psi_1) + \ldots$$

As in the case of the flow around a point of separation [1-4], or the flow around a plate trailing edge [7-10], the flow in domain 1 is described in a first approximation by linear supersonic flow theory. By using this theory, a relation can be obtained between the pressure perturbation and the vertical velocity component upward and downward from the zero streamline $\psi = 0$, whose Cartesian coordinate we denote by $Y_0{}^*$:

$$\sqrt{M_\infty^2 - 1}\, p_1(x_1, \psi_1 \to \pm 0) = \pm \frac{dY_0^*}{dx_1} \pm v_1(x_1, \psi_1 \to \pm 0).$$

The asymptotic expansions take the following form in domain 2:

$$x = \varepsilon^{3/4} x_2, \quad \psi = \varepsilon \psi_2, \quad p = \frac{1}{\gamma M_\infty^2} + \varepsilon^{1/2} p_2(x_2, \psi_2) + \ldots,$$

$$\rho = \rho_{20}(\psi_2) + \varepsilon^{1/2} \rho_2(x_2, \psi_2) + \ldots, \quad u = u_{20}(\psi_2) + \varepsilon^{1/2} u_2(x_2, \psi_2) + \ldots,$$

$$v = \varepsilon^{1/2} v_2(x_2, \psi_2) + \ldots, \quad h = h_{20}(\psi_2) + \varepsilon^{1/2} h_2(x_2, \psi_2) + \ldots,$$

$$y = \varepsilon y_{20}(\psi_2) + \varepsilon^{5/4} y_{21}(x_2, \psi_2) + \ldots$$

The first terms in the expansions $u_{20}(\psi_2)$, $\rho_{20}(\psi_2)$, $h_{20}(\psi_2)$, $y_{20}(\psi_2)$ are determined for $\psi_2 > 0$ by merger with the solution of the boundary layer equations before the interaction domain on the upper side of the profile trailing edge, and for $\psi_2 < 0$ by merger with the solution obtained for the boundary layer on the lower side of the profile. The flow in domain 2 turns out to be inviscid [1-4] and, in a first approximation, does not influence the pressure distribution in the interaction domain. The absence of a transverse pressure drop in domain 2 permits utilization of the pressure distribution found in domain 1 to compute the flow in the viscous near-wall layer (see [1-4]). The asymptotic representations and equations that describe the flow in domain 3 have the form

$$x = \varepsilon^{3/4} x_3, \quad \psi = \varepsilon^{3/2}\psi_3, \quad p = \frac{1}{\gamma M_\infty^2} + \varepsilon^{1/2} p_3(x_3, \psi_3) + \ldots, \quad \rho = \rho_w + \varepsilon^{1/4}\rho_3(x_3, \psi_3), \tag{2}$$

$$u = \varepsilon^{1/4} u_3(x_3, \psi_3) + \ldots, \; v = \varepsilon^{3/4} v_3(x_3, \psi_3) + \ldots, \; h = h_w + \varepsilon^{1/4} h_3(x_3, \psi_3),$$

$$\mu = \mu_w + \ldots, \; y = \varepsilon^{5/4} y_3(x_3, \psi_3) + \ldots;$$

$$\rho_w u_3 \frac{\partial u_3}{\partial x_3} + \frac{\partial p_3}{\partial x_3} = \rho_w u_3 \frac{\partial}{\partial \psi_3}\left(\rho_w \mu_w u_3 \frac{\partial u_3}{\partial \psi_3}\right), \quad \frac{\partial p_3}{\partial \psi_3} = 0, \quad \frac{\partial y_3}{\partial x_3} = \frac{v_3}{u_3}, \tag{3}$$

$$\frac{\partial y_3}{\partial \psi_3} = \frac{1}{\rho_w u_3}, \quad \frac{\partial h_3}{\partial x_3} = \frac{1}{\sigma}\frac{\partial}{\partial \psi_3}\left(\rho_w \mu_w u_3 \frac{\partial h_3}{\partial \psi_3}\right).$$

Equations (3) show that the flow in the viscous sublayer is described by the usual incompressible boundary-layer equations. However, the pressure gradient is not given here but should be determined during the solution by merging with the domain 1 since there is no transverse pressure drop in domain 2. The boundary layer on the upper and lower profile surfaces can, as has been noted above, have different friction stresses on the wall ($\psi_2 = \pm 0$) ahead of the interaction domain. Consequently, by using dimensionless friction stresses in the unperturbed boundary layer on the upper and lower sides of the profile $a_1 = \partial u_{20}/\partial y_{20}$ ($\psi_2 = +0$), $a_2 = \partial u_{20}/\partial y_{20}$ ($\psi_2 = -0$), in front of the interaction domain, boundary conditions can be obtained for the longitudinal velocity component in the viscous sublayer in the form

$$u_3 \to \sqrt{\frac{2a_1\psi_3}{\rho_w}} \text{ as } \psi_3 \to +\infty, \quad u_3 \to \sqrt{\frac{-2a_2\psi_3}{\rho_w}} \text{ as } \psi_3 \to -\infty.$$

Merging the vertical coordinates in domains 2 and 3, as is done in [4], yields the following expressions for the functions $y_{21}(x_2, +0)$:

$$y_{21}(x_2, +0) = \lim_{\psi_3 \to +\infty}\left(\int_0^{\psi_3} \frac{d\psi_3}{\rho_w u_3} - \sqrt{\frac{2\psi_3}{\rho_w a_1}}\right), \quad y_{21}(x_2, -0) = \lim_{\psi_3 \to -\infty}\left(-\int_{\psi_3}^{0} \frac{d\psi_3}{\rho_w u_3} + \sqrt{\frac{-2\psi_3}{\rho_w a_2}}\right).$$

This last merger of the vertical velocities and coordinates in domains 2 and 1 permits finding the pressure in the interaction domain, which defines the boundary value problem for domain 3 completely:

$$\sqrt{M_\infty^2 - 1}\, p_3(x_3, +0) = \frac{d}{dx_3}\left[Y_0^*(x_3, +0) + \lim_{\psi_3 \to +\infty}\left(\int_0^{\psi_3} \frac{d\psi_3}{\rho_w u_3} - \sqrt{\frac{2\psi_3}{\rho_w a_1}}\right)\right],$$

$$\sqrt{M_\infty^2 - 1}\, p_3(x_3, -0) = \frac{d}{dx_3}\left[-Y_0^*(x_3, -0) + \lim_{\psi_3 \to -\infty}\left(\int_{\psi_3}^{0} \frac{d\psi_3}{\rho_w u_3} - \sqrt{\frac{-2\psi_3}{\rho_w a_2}}\right)\right].$$

Let us note that $x_3 > 0$ $p_3(x_3, +0) = p_3(x_3, -0)$ and $Y_0^*(x_3, +0) = Y_0^*$ $(x_3, -0)$ for $x_3 > 0$ while the Cartesian coordinate of the zero streamline agrees with the upper and lower profile surfaces, respectively, and $p_3(x_3, +0) \neq p_3(x_3, -0)$ for $x_3 < 0$. It is convenient to use the boundary layer equations (3) in the (x_3, y_3) variables and the following change of variables and flow functions:

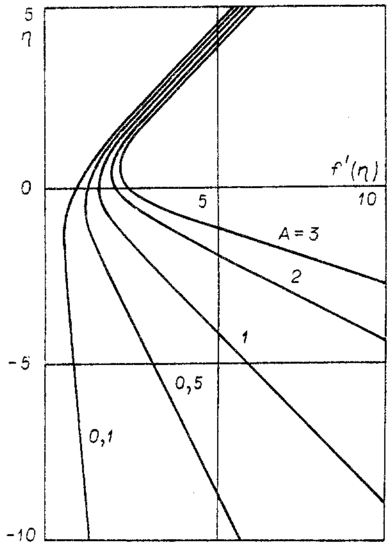

Fig. 1

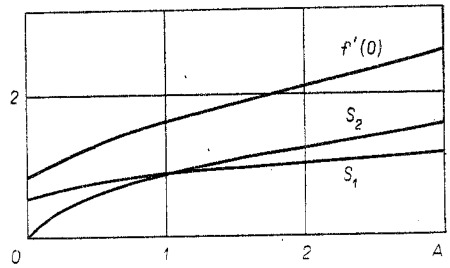

Fig. 2

$$
\begin{aligned}
&u_3 = \left(\mu_w a_1/\rho_w^2 (M_\infty^2 - 1)^{1/2}\right)^{1/4} U, \quad v_3 = \left(\mu_w^3 a_1^3 (M_\infty^2 - 1)^{1/2} \rho_w^{-2}\right)^{1/4} V,\\
&x_3 = \left(\mu_w a_1^5 \rho_w^2 (M_\infty^2 - 1)^{3/2}\right)^{-1/4} X, \quad y_3 = \left(\mu_w/a_1^3 \rho_w^3 (M_\infty^2 - 1)^{1/2}\right)^{1/4} Y,\\
&p_3 = \left(\mu_w a_1/(M_\infty^2 - 1)^{1/2}\right)^{1/2} P, \quad Y_0^* = \left(\mu_w/a_1^3 \rho_w^3 (M_\infty^2 - 1)^{1/2}\right)^{1/4} \overline{Y}_0
\end{aligned} \tag{4}
$$

to investigate the flow in domain 3. Assuming the profile shape in the neighborhood of the trailing edge to be a wedge with apex half-angle θ at the angle of attack α to the free stream direction, we obtain the following boundary value problem for a viscous sublayer:

$$
\begin{gathered}
U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{dP}{dX} + \frac{\partial^2 U}{\partial Y^2}, \quad \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad P = \begin{cases} P_v, & Y > 0, \\ P_f, & Y < 0, \end{cases}\\
\frac{\partial U}{\partial Y} \to 1 \ \text{as} \ Y \to +\infty, \quad \frac{\partial U}{\partial Y} \to -A \ \text{as} \ Y \to -\infty,
\end{gathered} \tag{5}
$$

$$
U = \frac{Y + |Y|}{2} - \frac{Y - |Y|}{2}A, \quad P_v \to -\theta - \alpha, \quad P_f \to -\theta + \alpha \ \text{as} \ X \to -\infty,
$$

$$
U = V = 0 \ \text{as} \ Y = 0, \quad P_v = -\theta - \alpha + \frac{d\Delta}{dX}, \quad P_f = -\theta + \alpha + \frac{d\Delta_1}{dX} \ \text{as} \ X \leqslant 0,
$$

$$
V = 0 \ \text{as} \ Y = 0, \quad P_v = \frac{d\overline{Y}_0}{dX} + \frac{d\Delta}{dX}, \quad P_f = -\frac{d\overline{Y}_0}{dX} + \frac{d\Delta_1}{dX}, \quad P_v = P_f \ \text{as} \ X > 0,
$$

$$
P \to 0 \ \text{as} \ X \to +\infty, \quad \Delta = \lim_{Y \to +\infty} (Y - U), \quad \Delta_1 = \lim_{Y \to -\infty} \left(-Y - \frac{U}{A}\right),
$$

where $A = a_2/a_1$.

The boundary value problem formulated here for θ = 0 and A = 1 goes over into a problem examined in detail in [8]. The condition θ = 0 means that the profile has a zero streamline in the neighborhood of the trailing edge, or the profile thickness is less in order of magnitude than $\varepsilon^{5/4}$. The parameter A becomes one when the friction stresses are equal on the upper and lower sides of the profile trailing edge. This is possible both in the case of a flat plate [8], and in the flow around a symmetric profile at zero angle of attack ($F_1(X_0) \equiv F_2(X_0)$). In contrast to [10], utilization of the similarity transform (4) permitted elimination of the free-stream Mach number from the formulation of the boundary value problem (5). In fact, as is shown in [10], not only the ratio of the friction stresses in the unperturbed boundary layer, but also the free stream Mach number enters as one of the boundary conditions in the formulation of the problem for the viscous sublayer in supersonic flow over the trailing edge of a flat plate with different velocities on its opposite sides.

Let us examine the asymptotic behavior of the solution of the boundary value problem (5) obtained for large values of X. By using the continuity equation we introduce the stream

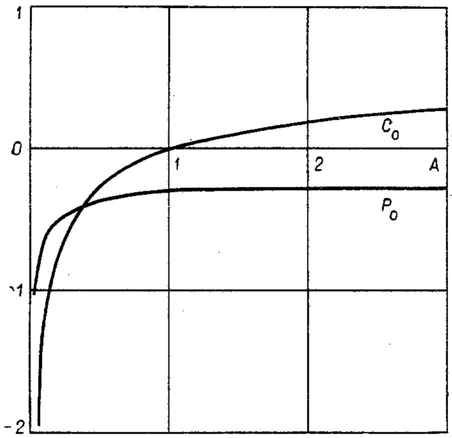

Fig. 3

function ψ and, following [8], assume that the solution can be sought as $X \to +\infty$ in the form

$$P = P_0 X^{-2/3} + \ldots, \quad \psi = X^{2/3} f_0(\eta) + X^{1/3} f_1(\eta) + \ldots,$$

where the self-similar variable is $\eta = Y/X^{1/3}$. Then, we obtain an equation and boundary conditions for the first term of the stream-function expansion

$$f_0''' + \frac{2}{3} f_0'' f_0 - \frac{1}{3}(f_0')^2 = 0, \quad f_0''(+\infty) = 1, \quad f_0''(-\infty) = -A. \tag{6}$$

In the case of supersonic flow around a flat plate at a small angle of attack, which was investigated in [8], the boundary layer on the upper and lower sides of the plate trailing edge has identical friction stress. Hence, A = 1, and the last boundary condition in (6) would become $f_0''(-\infty) = -1$. As has been shown in [5], this boundary condition can be replaced by $f_0''(0) = 0$, and the solution of (6) can be sought only for positive values of η.

It is found in [5] in solving this boundary value problem that

$$f_0'(0) = 1.61, \quad f_0' \to \pm\eta - 3P_0 \quad \text{as} \quad \eta \to \pm\infty, \quad P_0 = -0.297.$$

In the nonsymmetric case that is examined in this paper ($A \neq 1$), it is necessary to solve (6) for the whole range $-\infty < \eta < +\infty$. The boundary conditions (6) permit indicating the behavior of $f_0'(\eta)$ for large values of the variable η:

$$f_0'(\eta) \to \eta + S_1 \quad \text{as} \quad \eta \to +\infty, \quad f_0'(\eta) \to -\eta A + S_2 \quad \text{as} \quad \eta \to -\infty,$$

where S_1 and S_2 are constants determined from the solution of the boundary value problem (6).

This affords the possibility of finding the first terms of the expansions for the pressure and the Cartesian coordinate of the streamline as $X \to +\infty$:

$$P = P_0 X^{-2/3} + \ldots, \quad \bar{Y}_0 = C_0 X^{1/3} + \ldots, \quad P_0 = \frac{1}{6}\left(-S_1 - \frac{S_2}{A}\right), \quad C_0 = \frac{1}{2}\left(S_1 - \frac{S_2}{A}\right).$$

The boundary value problem (6) in which the single parameter A enters was solved numerically for the values $0 \leqslant A \leqslant 3.5$. The behavior of the first derivative $f_0'(\eta)$ is displayed in Fig. 1 for different values of A. The curves displaying the dependence of the velocity at the zero streamline and the constants S_1 and S_2 on the magnitude of the parameter A are presented in Fig. 2. In particular, results are obtained for the case A = 1, which are in good agreement with the data in [5]:

$$f_0'(0) = 1.610, \quad S_1 = S_2 = 0.894, \quad P_0 = -0.298, \quad C_0 = 0.$$

In this case (A = 1), the first term in the expansion for the Cartesian coordinate of the zero streamline is missing ($C_0 = 0$), and the expansion starts with the next term whose form is examined in [8]. However, for $A \neq 1$ the Cartesian coordinate of the zero streamline behaves as $C_0 X^{1/3}$ as $X \to +\infty$, where the quantity C_0 depends only on the parameter A. Curves displaying the dependence of the constants P_0 and C_0 in the first terms of the expansions for the pressure and the Cartesian coordinate of the zero streamline on A are represented in

Fig. 3. For A > 1 the constants P_0 remain negative and increase slowly simultaneously with the growth of the parameter A. Thus for A increasing from 1 to 1.5, P_0 increases by only 5% and is practically invariant as A grows further. However, as A diminishes from 1 to 0, the absolute value of P_0 starts to grow. Thus, for A = 0.1, the quantity P_0 diminishes by more than twice as compared with its value for A = 1. This results in slower damping of the pressure perturbation as $X \to +\infty$ and A < 1 whereupon the extent of the interaction domain downstream from the profile trailing edge increases by comparison to the case A = 1. Computations also showed that the constant C_0 characterizing the deviation of the zero streamline from the abscissa axis of the Cartesian (X_0, Y_0) coordinate system varies considerably more rapidly for A < 1 than for A > 1. Such a difference in the behavior of the quantities P_0 and C_0 for A < 1 and A > 1 is explained by the following. Upon the insertion of new variables (4), the magnitude of the dimensionless friction stress on the upper side of the profile trailing edge α_1 was used, and the magnitude α_2 here entered only into the parameter $A = \alpha_2/\alpha_1$. The length of the interaction domain is, as follows from (4), proportional to the dimensionless friction stress to the power $m = -5/4$. The quantity P_0 and the interaction domain length are determined principally by the least dimensionless friction stress in the unperturbed boundary layer. Hence, in the variables in which the problem (5) is formulated, the constant P_0 and the extent of the interaction domain downstream of the profile trailing edge are practically invariant as the parameter A increases for $A = \alpha_2/\alpha_1 > 1$ since they are defined by a quantity whose influence has already been taken into account upon insertion of the variables (4). For A < 1 downstream propagation of the perturbations and the flow in the wake depend principally on the magnitude of the friction stress α_2, and the dependence on the parameter A in the variables (4) becomes stronger. Let us note that for A = 0 boundary layer separation occurs on the lower side of the profile upstream of the interaction domain being formed at the profile trailing edge. In the neighborhood of the point of separation, an interaction domain here occurs that has been examined in detail in [1-4]. In this case the flow at the trailing edge can be reconstructed, and the asymptotic expansions and flow diagrams proposed here can become incorrect.

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